perpendiculars from F and F' upon the normal MT and by considering two pairs of similar triangles, it is easily seen from a figure that

$$(2) \qquad \frac{n - \delta \cos \omega}{\delta' \cos \omega - n} = \frac{\delta}{\delta'} \quad \text{or} \quad \frac{1}{\delta} + \frac{1}{\delta'} = \frac{2 \cos \omega}{n}.$$

From (1) and (2) there results at once

$$(3) \qquad R = \frac{n}{\cos^2 \omega},$$

the formula mentioned above.

If the curvature of the curve Γ is such that the point F' lies on the side opposite to F, then by a very slight modification of the derivation given by Humbert it may be shown that (1) is again true provided that δ' be regarded as negative. In this case there is no caustic by reflection in the physical sense and it is not considered by Humbert. Also a proof similar to that indicated above shows that (2) also is true if δ' be regarded as negative. Thus (3) is true for both cases. It is thus somewhat more general and at the same time simpler than (1); it also has the advantage of being more convenient for geometrical constructions. Thus, if the point O has been found on the evolute of a curve Γ corresponding to the point M of Γ, the formula (3) gives an easy construction for the point F' on the caustic of Γ with respect to a given point F. If, for example, the curve Γ is a circle the caustic may be easily traced by this construction. If, on the other hand, Γ is a conic and F is a focus, then γ reduces to the other focus, or to a point at infinity in the case of a parabola, and the formula gives the construction given by Professor da Cunha.

Another construction for the center of curvature may be derived from a property of triangles. In any triangle FMF' let C be the middle point of the side FF', T the point in which the bisector of the angle at M meets FF', P the point in which the side FM is cut by the perpendicular to the bisector at T, O the point in which the perpendicular to FM at P meets the bisector MT (O is the center of curvature when F, F' and M have the meaning above) and finally Q the point in which TP meets CM. The property referred to is the fact that QO is perpendicular to FF'. A similar theorem is true for the external bisector. The second construction, which applies to the general case as well as to conics, is then as follows: Erect a perpendicular to the normal at the point where it cuts the axis and from the point in which this perpendicular meets the line joining the center with the given point on the curve drop a perpendicular to the axis and produce it to meet the normal. This point is the center of curvature.

PROBLEMS FOR SOLUTION.

2829. Proposed by E. S. PALMER, New Haven, Conn.

Given a set of arbitrary pairs of positive integers (a_p, b_p), $(p = 1, 2, \cdots, n)$: (a). Is it always possible to find a set of positive integers k_p, $(p = 1, 2, \cdots, n)$ such that

$$k_p a_p + k_p b_p > \sum_{r=1}^{r=n} k_r a_r, \qquad (p = 1, 2, 3, \cdots, n).$$

(b) If or when possible, show how to find k_p?

2830. Proposed by WILLIAM HOOVER, Columbus, Ohio.

An elastic string connects a pair of opposite vertices of a square whose sides are four equal rods freely jointed, each of length $2a$. The system is suspended vertically from a vertex attached to the string and is at rest. If the string be cut, and θ is the acute angle any side makes with the vertical at any moment during the motion, determine the angular velocity of any rod.

2831. Proposed by B. J. BROWN, Kansas City, Mo.

Given that the series $\sum_0^\infty a_n y^n$ is absolutely convergent when $|y| < 1$, prove that the series $\sum_0^\infty a_n(2x \cos \theta - x^2)^n$ may be arranged in powers of x provided $|x| < \frac{2}{5}$.

Prove that if x lies between 1 and 2, the series $\sum_0^\infty (2x - x^2)^n$ is convergent and consists only of positive terms, but that the series obtained by arranging it in powers of x, diverges.

2832. Proposed by S. A. COREY, Des Moines, Iowa.

Prove that the square of the sum of four squares is the sum of four squares, that the square of the sum of eight squares is the sum of six squares, and that the square of the sum of sixteen squares is the sum of ten squares.

2833. Proposed by W. H. ECHOLS, University of Virginia.

In *Engineering,* London, during 1917 appeared the following equations concerning the stability of ships; they are employed by the naval constructors in the Norfolk, Virginia, Navy Yard, and they are of importance,

$$\tan^3 \theta_0 + \frac{2m}{\rho} \tan \theta_0 - \frac{2x_0}{\rho} = 0,$$

$$\tan^3 (\theta_0 + \theta_1) + \frac{2m}{\rho} \tan (\theta_0 + \theta_1) - \frac{2x_0}{\rho} - \frac{2x}{\rho} = 0,$$

$$\tan^3 (\theta_0 - \theta_2) + \frac{2m}{\rho} \tan (\theta_0 - \theta_2) - \frac{2x_0}{\rho} + \frac{2x}{\rho} = 0.$$

The required unknowns are θ_0, x_0 and m. The constants have values as follows, θ_1 and θ_2 are positive angles ranging from $15'$ to $10°$, x is positive and less than 5, and ρ is positive with considerable range of values. A rapid solution involving small labor is desired, determining x_0 within the same limits as given for x.

SOLUTIONS OF PROBLEMS.

2740 [1919, 35]. Proposed by E. W. CHITTENDEN, University of Illinois.

Establish the identity (the determinant is of order n):

$$\begin{vmatrix} a-\lambda, & a, & a, & \cdots, & a \\ a, & a-\lambda, & a, & \cdots, & a \\ a, & a, & a-\lambda, & \cdots, & a \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ a, & a, & a, & \cdots, & a-\lambda \end{vmatrix} = (-1)^{n-1}\lambda^{n-1}(na - \lambda).$$

SOLUTION BY L. E. MENSENKAMP, Freeport, Illinois.

We proceed to establish this relation by mathematical induction. The relation is obviously true for $n = 2$. Assume it is true for a determinant of order n as written above by the proposer. Multiply both sides of this equality by $\dfrac{-\lambda(na + a - \lambda)}{na - \lambda}$. The right-hand member of the